A Life Expectancy Study based on the Deterioration Function and an Application to Halley's Breslau Data

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Abstract: Further to the proposal and application of a stochastic methodology and the resulting first exit time distribution function to life table data we introduce a theoretical framework for the estimation of the maximum deterioration age and to explore on how "vitality," according to Halley and Strehler and Mildvan, changes during the human lifetime. The mortality deceleration or mortality leveling-off is also explored. The effect of the deterioration over time is estimated as the expectation that an individual will survive from the deterioration caused in his organism by the deterioration mechanism. A method is proposed and the appropriate software was developed for the estimation of life expectancy. Several applications follow.
The method was applied to the Halley life table data of Breslau. Extrapolations are done showing a gradual improvement of vitality mechanisms during last centuries.
Keywords: Life expectancy, Life expectancy at birth, Deterioration function, Late-Life Mortality Deceleration, Mortality Leveling-off, Mortality Plateaus Vitality, Halley Breslau data.

Introduction

The methods and techniques related to life expectancy started to grow from the proposal of Life Tables' system by Edmond Halley (1693) in his study on Breslau birth and death data. The related methods include mathematical models starting from the famous Gompertz (1825) model. The Gompertzian influence on using the mortality data to construct life tables was more strong than expected. He had proposed a model and a method to cope with the data. The model was relatively simple but quite effective because he could, by applying his model, to account for the main part of the data set. The method he proposed and applied was based on a data transformation by adding, dividing and finally taking logarithms of the raw data in an approach leading to a linearization of the original distribution. This was very important during Gompertz days when the calculations where very laborious. Furthermore, this method of data linearization made possible the wide dissemination, approval and use by the actuarial people of the Gompertz model and the later proposed Gompertz–Makeham (Makeham, 1860) variation. However, the use of the logarithmic transformation of the data points until nowadays turned to be a strong drawback in the improvement of the related fields. By taking logarithms of a transformed form of the data it turns to have very high absolute values for the data points at the beginning and at the end of the time

Paper Version: October 13, 2011,
October 01, 2011 version at: http://arxiv.org/abs/1110.0130

interval, the later resulting in serious errors due to the appearance of very high values at high ages. To overcome these problems resulting from the use of logarithmic transformations several methods and techniques have being proposed and applied to the data thus making more complicated the use of the transformed data. The approaches to find the best model to fit to the transformed data leaded to more and more complicated models with the Heligman-Pollard (1980) 8-Component Model and similar models to be in use today. The task was mainly to find models to fit to data well, instead to search for models with a good explanatory ability. Technically the used methodology is directed to the actuarial science and practice and to applications in finance and insurance (see the method proposed by Lee and Carter, 1992).

Another quite annoying thing by applying logarithms is that the region around the maximum point of the raw data death distribution including this inflection point along with the left and right inflection points are all on the almost straight line part of the logarithmic curve.

From the modelling point of view it is quite difficult to find from the transformed data the characteristics of the original data distribution. Even more when constructing the life tables the techniques developed require the calculation of the probabilities and then the construction of a "model population" usually of 100000 people. The reconstruction of the original data distribution from the "model population" provides a distribution which is not so-close to the original one.

Here we propose a method to use models, methods and techniques on analysing data without transforming the data by taking logarithms. While this methodology has obvious advantages it faces the problem of introducing it in a huge worldwide system based on the traditional use of methods, models and techniques arising from the Gompertzian legacy. To cope with the well established life table data analyses we present our work as to test the existing results, to give tools for simpler applications and more important to make reliable predictions and forecasts. We give much attention to the estimation and analysis of the life expectancy and the life expectancy at birth as are the most important indicators for policy makers, practitioners and of course researchers from various scientific fields.

The Deterioration Function: Further analysis

The Deterioration Function for the model $H(t)=l-(bt)^c$ or for the model $H(t)=(bt)^c$ is expressed by the following formula (Skiadas, 2011). This formula provides the value of the curvature at every point ($H(t)$, t).

$$K(t)=\frac{|\,c(c-1)b^c t^{c-2}\,|}{(1+c^2b^{2c}t^{2c-2})^{3/2}}$$

This is a bell-shaped distribution presented in Figure 1. The first exit time IM-model (Skiadas, 2007, 2010a, 2010b, 2011 and Janssen and Skiadas, 1995) is applied to the female mortality data of Italy for the year 1950.

$$g(t) = \frac{k(l + (c-1)(bt)^c)}{\sqrt{t^3}} e^{-\frac{(l-(bt)^c)^2}{2t}}$$

A simpler 3 parameter version of this model arises when the infant mortality is limited thus turning the parameter l to be: l=0 and the last formula takes the simpler form:

$$g(t) = \frac{k(c-1)(bt)^c}{\sqrt{t^3}} e^{-\frac{(bt)^{2c}}{2t}}$$

The data, the fitting and the deterioration curves are illustrated in Figure 1. The deterioration function starts from very low values at the first stages of the lifetime and is growing until a high level and then gradually decreases. As the main human characteristics remain relatively unchanged during last centuries it is expected that the deterioration function and especially the maximum point should remain relatively stable in previous time periods except of course of the last decades when the changes of the way of living and the progress of biology and medicine tend to shift the maximum deterioration point to the older age periods.

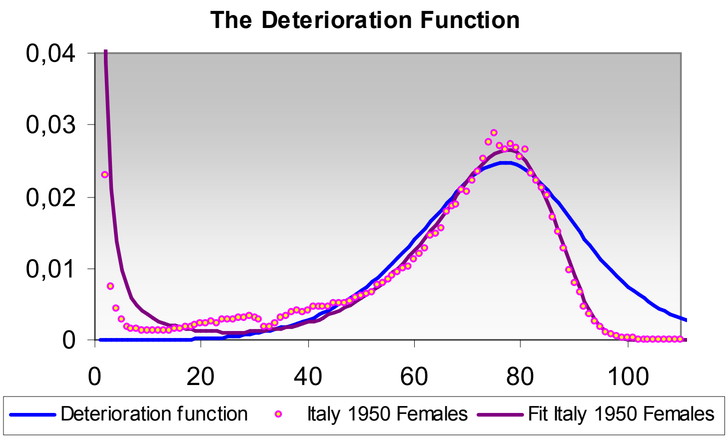

Fig. 1. Deterioration function, raw and estimated data for Italy.

We can also explore the Greenwood and Irwin (1939) argument for a late-life mortality deceleration or the appearance of mortality plateaus at higher ages by observing the shape of the deterioration function. As we can see in Figures 1 and 2 the deterioration tends to decrease in higher ages and especially after reaching the maximum value, leading to asymptotically low levels thus explaining what we call as Mortality Leveling-off. The deterioration of the organism tends to zero at higher ages. Economos (1979, 1980) observed a mortality leveling-off in animals and manufactured items.
The characteristic non-symmetric bell-shaped form of the deterioration function is illustrated in Figure 2 for females in France and for various periods from 1820 to 2007. The IM-model is applied to the data provided by the human mortality database for France in groups of 10 years. The form of the deterioration function is getting sharper as we approach resent years. The improvement of the way of living is reflected in the left part of the deterioration function. The graph for 1820-1829 presents the deterioration starting from very early ages. Instead the graph for the period 1900-1909 shows an improvement in the early deterioration period. The improvement continues for 1950-1959 and 2000-2007 periods.

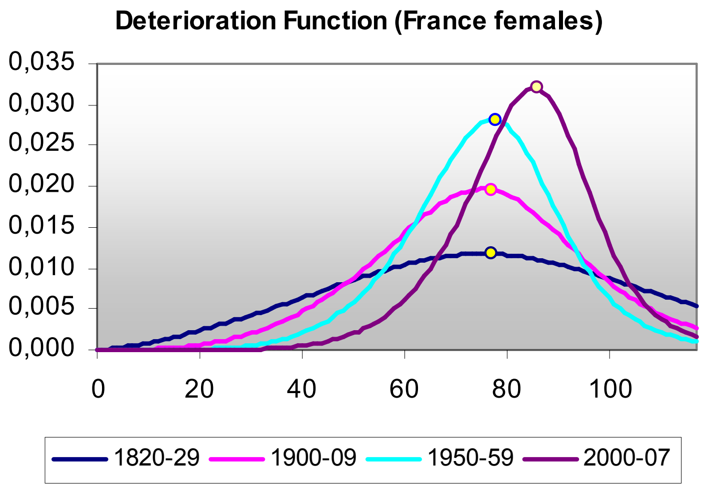

Fig. 2. Deterioration function for females in France

However, the maximum point of these curves is achieved at almost the same year of age from 1820-1829 until the 1950-1959 period. This is an unexpected result as it was supposed that the general improvement of the way of living would result in a delay in the deterioration mechanisms. To clarify this observation we will further analyze the deterioration function.

A main characteristic of the deterioration function is its maximum point achieved at:

$$T_{Deter} = \left[\frac{(c-2)}{(2c-1)c^2 b^{2c}} \right]^{\frac{1}{2c-2}}$$

We will apply this formula to mortality data of several countries and for various time periods. The task is to explore our argument for the stability of the maximum deterioration point around certain age limits and of a shift of this point to higher ages.

Figure 3 for females in France from 1816 to 2007 is quite interesting. Four graphs are illustrated. The data are summarized in Table I.

TABLE I

Life expectancy at birth and Maximum deterioration age

France, females

Year	HM-database	IM-model	3p-model	Max Det
1816-1819	39,54	34,69		74,79
1820-1829	39,58	34,98		75,20
1830-1839	39,66	37,39		75,36
1840-1849	41,19	39,15		75,67
1850-1859	40,43	39,67		75,43
1860-1869	42,27	39,74		75,11
1870-1879	42,33	41,51		75,63
1880-1889	44,81	42,54		75,45
1890-1899	46,72	45,77		74,98
1900-1909	49,83	48,87		75,23
1910-1919	51,53	54,06		75,53
1920-1929	56,41	55,41		75,79
1930-1939	60,89	60,13		76,06
1940-1949	62,00	61,50		76,37
1950-1959	71,16	66,68	73,72	77,15
1960-1969	74,59	71,60	75,78	78,61
1970-1979	76,89	75,25	77,54	79,86
1980-1989	79,45	78,75	80,04	81,24
1990-1999	81,81	81,96	82,70	83,51
2000-2007	83,47	82,87	83,27	85,26

The data for the life expectancy at birth (blue line) are collected from the human mortality database for France (females in 10 year groups). The life expectancy at birth estimates based only on the fitting estimations of the IM model (based on the death data only) is presented with a magenta line. As the infant mortality becomes negligible during last decades we can estimate the life expectancy at birth by based on a 3-parameters alternative of the IM-model (red line). Note that according to the Nathan Keyfitz (2005) theory for a stable population the life expectancy estimates tend to coincide when using different methods. Here the three different estimates for the life expectancy at birth are similar for the last 20 years. That is interesting is that the life expectancy at birth is approaching the maximum deterioration age (light blue line) which may be seeing as a plateau for the life expectancy at birth.

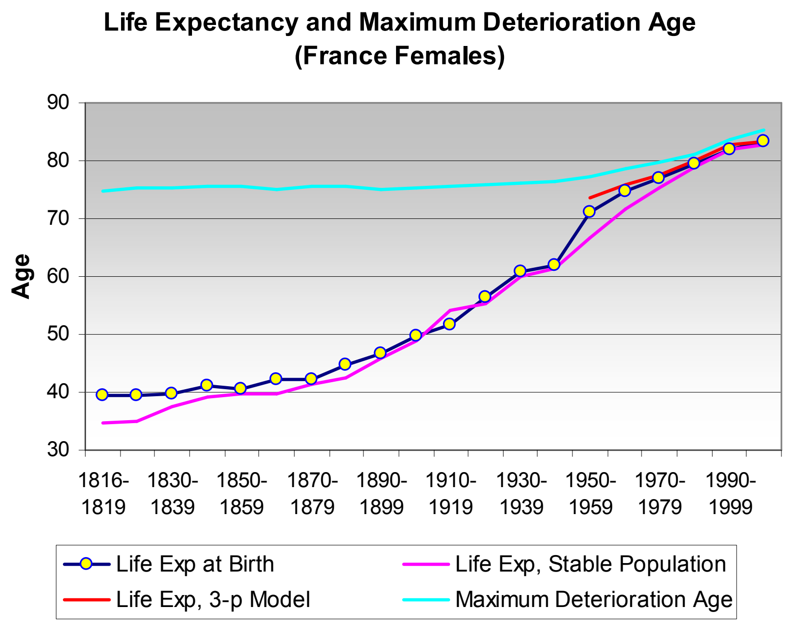

Fig. 3. Life expectancy at birth and maximum deterioration age for France.

Figure 4 illustrates the life expectancy at birth (blue line) with data collected from the human mortality database (females in 10 year groups). As the infant mortality becomes negligible during last decades we can estimate the life expectancy at birth by based on a 3-parameters alternative of the IM-model (red line). The maximum deterioration age is expressed by a cyan line. The results for the four countries studied, A) Netherlands, B) Denmark, C) Italy and D) Norway are similar to the previous application for France.

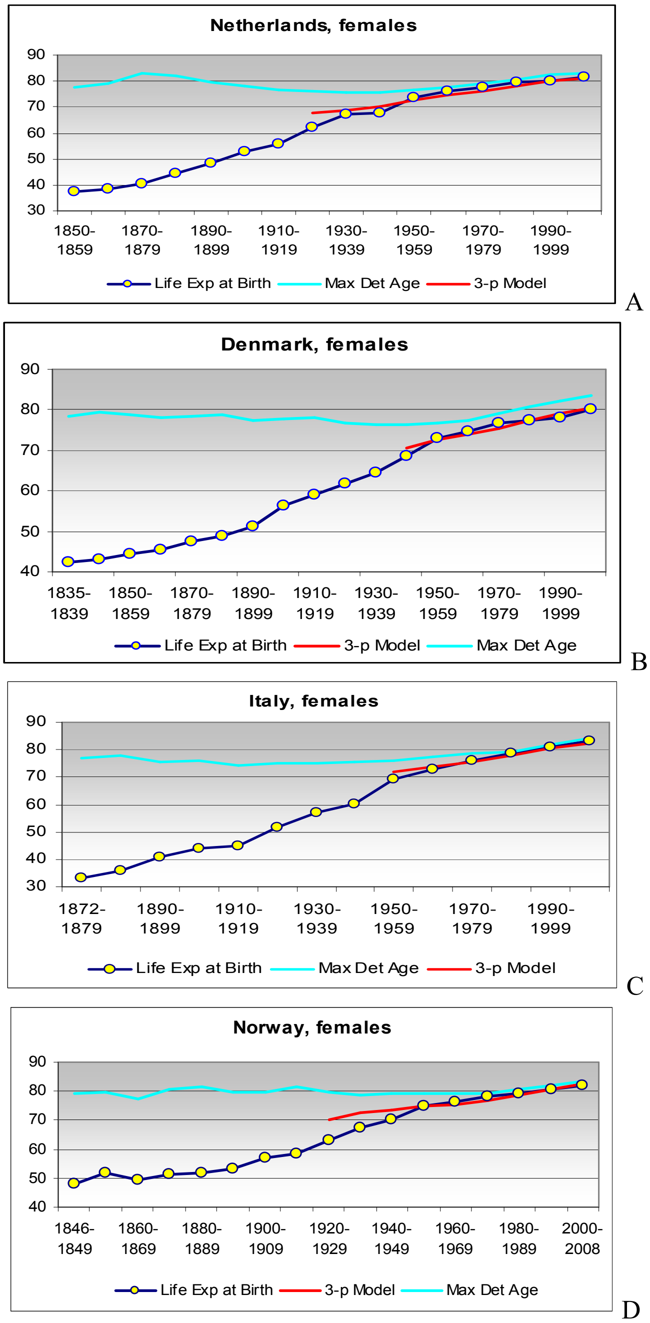

Figure 4. Life expectancy at birth and maximum deterioration age for A) Netherlands, B) Denmark, C) Italy and D) Norway.

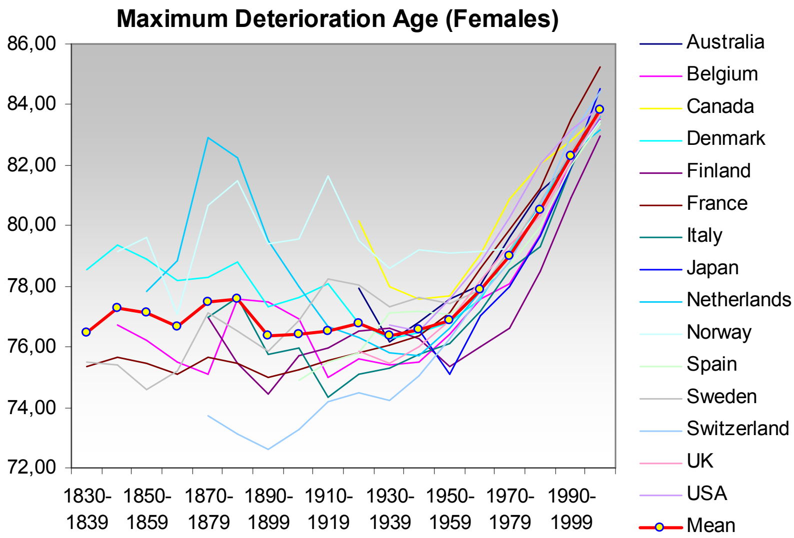

Fig. 5. Maximum deterioration age for 15 countries and mean value.

The age year where the maximum value for the deterioration function is achieved for females in various countries and for several time periods is illustrated in Figure 5. The data for females from various countries for 10 year periods from the human mortality data base are used and the Infant Mortality First Exit Density Model (IM-model) is applied. The maximum deterioration age for females was between 72 – 84 years from 1830 to 1950 for the countries studied (Table II). For all the cases a continuous growth appears for the maximum deterioration age after 1950 until now. That is more important is that the mean value of the maximum deterioration age was between 76 and 78 years for 140 years (1830-1970) irrespective of the fluctuations in the life expectancy supporting the argument for an aging mechanism in the human genes. However, the scientific and medical developments after 1950 gave rise in a gradual increase of the level of the maximum deterioration age from 77 to 84 years in the last 60 years (1950-2010).

TABLE II

The Maximum Deterioration Age in 15 Countries (Females)

Year	Australia	Belgium	Canada	Denmark	Finland	France	Italy	Japan	Netherlands	Norway	Spain	Sweden	Switzerland	UK	USA	Mean
1830-1839				78,54		75,36						75,50				76,47
1840-1849		76,70		79,35		75,67				79,15		75,42				77,26
1850-1859		76,20		78,91		75,43			77,85	79,61		74,61				77,10
1860-1869		75,50		78,18		75,11			78,86	77,07		75,21				76,65
1870-1879		75,10		78,29	76,98	75,63	76,99		82,88	80,70		77,12	73,70			77,49
1880-1889		77,60		78,79	75,46	75,45	77,65		82,25	81,48		76,52	73,14			77,59
1890-1899		77,50		77,32	74,44	74,98	75,76		79,51	79,42		75,87	72,61			76,38
1900-1909		76,90		77,64	75,69	75,23	75,95		77,96	79,55	74,88	76,86	73,26			76,39
1910-1919		74,99		78,11	75,98	75,53	74,34		76,68	81,63	75,48	78,25	74,17			76,52
1920-1929	77,94	75,60	80,15	76,76	76,51	75,79	75,11		76,34	79,52	75,78	78,01	74,49	75,87		76,76
1930-1939	76,14	75,40	78,00	76,25	76,64	76,06	75,29		75,81	78,61	77,11	77,34	74,21	75,47	76,71	76,36
1940-1949	76,82	75,50	77,60	76,47	76,26	76,37	75,76	76,57	75,69	79,22	77,16	77,61	75,03	75,99	76,52	76,57
1950-1959	77,56	76,40	77,68	76,81	75,36	77,15	76,10	75,10	76,60	79,09	77,05	77,43	76,24	76,89	77,59	76,87
1960-1969	78,04	77,60	79,03	77,57	76,02	78,61	77,15	77,00	77,84	79,14	77,92	77,87	77,71	78,20	78,82	77,90
1970-1979	79,59	78,10	80,88	79,12	76,60	79,86	78,54	77,97	78,99	79,27	78,69	79,19	78,78	79,38	80,29	79,02
1980-1989	81,14	79,70	82,04	80,79	78,51	81,24	79,30	79,67	80,80	80,55	79,99	80,78	80,64	80,29	82,06	80,50
1990-1999	82,07	82,20	82,79	82,22	80,95	83,51	81,88	81,93	82,38	82,16	82,00	82,47	82,96	82,10	83,13	82,32
2000-2007	83,59	83,50	83,70	83,64	82,96	85,26	83,97	84,54	83,18	83,47	83,37	83,96	84,41	83,66	83,92	83,81

Stability of the Deterioration Function Characteristics

The derivation of the deterioration function of a population allows us to make early estimates for the life expectancy. The main assumption is based on accepting a theory for an internal deterioration mechanism driven by a code which governs the life expectancy. If this assumption holds the deterioration function should include information for the future life expectancy even when using data from periods when the mean life duration was relatively small. In the previous chapter we have found that the maximum value of the deterioration function in many countries was set at high levels even when dealing with mortality data coming from various countries and from the last two centuries.
The next very important point is to estimate the total effect of the deterioration to a population in the course of the life time termed as *DTR*. This is expressed by the following summation formula:

$$DTR = \int_0^t tK(t)dt \approx \sum_0^t tK(t)$$

Where t is the age and $K(t)$ is the deterioration function.
The last formula expresses the expectation that an individual will survive from the deterioration caused in his organism by the deterioration mechanism. The result is given in years of age in a Table like the classical life tables. The deterioration function is estimated from 0 to 117 years a limit set according to the existing death data sets. The estimated life expectancy levels are not the specific levels at the dates of the calculation but refer to future dates when the external influences, illnesses and societal causes will be reduced to a minimum following the advancement of our population status. The life expectancy levels seem to be reached in the recent years in some countries and in the forthcoming decades for others.
DTR will be a strong indicator for the level of life expectancy of a specific population mainly caused by the DNA and genes. Due to its characteristics DTR can also be estimated from only mortality data (number of deaths per age or death distribution) thus making simpler the handling of this indicator even when population data are missing or are not well estimated. Another important point of the last formula is that we can find an estimator of the life expectancy in various age periods and to construct a life table. As it is expected the existence of a deterioration law will result in a population distribution over time thus making possible the construction of life tables by using the population distribution resulting from the deterioration law.

As the introduction of the deterioration function and the DTR indicator are quite new terms introduced when using the stochastic modeling techniques and the first exit time or hitting time theory we have applied the DTR and other forms resulting from the deterioration function to the mortality data for countries included in the Human Mortality Database (HMD). A main advantage by using these data sets is that are systematically collected and developed as to be able to make applications and comparisons between countries. The death data for 10-year periods are preferred as to avoid local fluctuations. However, the results are also strong when using and the other data sets from HMD for 5-year of 1-year periods.

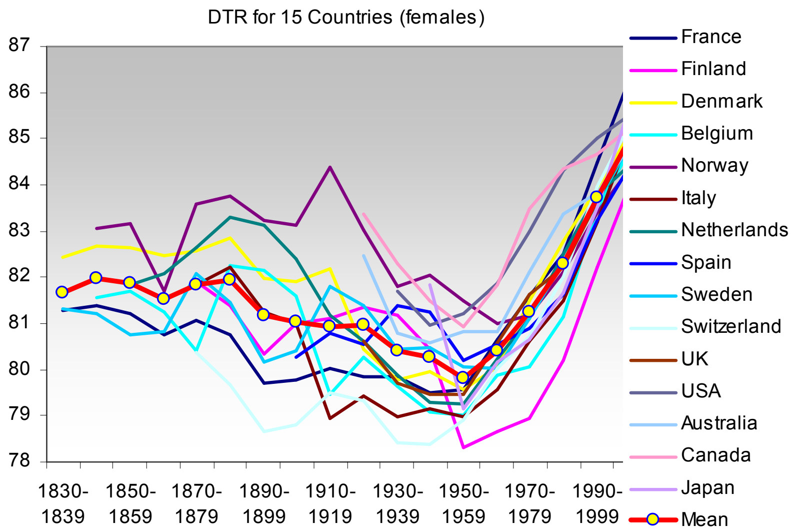

Fig. 6. DTR for 15 countries and mean value.

The DTR is estimated for 15 countries for large time periods. As it is presented in Figure 6 the DTR for 150 years from 1830 to 1980 was between 78 and 84 years of age with the mean value to be from 79.80 to 81.98 years (see Table III). The lowest value 79.80 years was achieved the period 1950-1959. The main conclusion is that the DTR can be used as a measure of the future life expectancy levels. An example is based on the estimates for Sweden from 1751. The same method can be applied for other countries.

TABLE III

The DTR effect in 15 Countries (Females)

Year	Australia	Belgium	Canada	Denmark	Finland	France	Italy	Japan	Netherlands	Norway	Spain	Sweden	Switzerland	UK	USA	Mean
1830-1839				82,43		81,29						81,30				81,67
1840-1849		81,55		82,68		81,39				83,07		81,20				81,98
1850-1859		81,70		82,65		81,20			81,84	83,16		80,76				81,89
1860-1869		81,23		82,47		80,75			82,07	81,71		80,81				81,51
1870-1879		80,41		82,57	81,92	81,07	81,85		82,62	83,59		82,07	80,37			81,83
1880-1889		82,25		82,86	81,37	80,76	82,22		83,30	83,77		81,44	79,66			81,96
1890-1899		82,16		81,98	80,34	79,70	81,23		83,12	83,22		80,15	78,67			81,17
1900-1909		81,61		81,89	81,01	79,77	80,98		82,38	83,13	80,26	80,42	78,82			81,03
1910-1919		79,45		82,20	81,10	80,01	78,93		81,18	84,39	80,78	81,81	79,49			80,93
1920-1929	82,47	80,28	83,38	80,43	81,35	79,86	79,42		80,63	83,02	80,54	81,37	79,32	80,61		80,98
1930-1939	80,80	79,64	82,30	79,77	81,16	79,86	78,98		79,88	81,80	81,40	80,45	78,42	79,72	81,70	80,42
1940-1949	80,58	79,09	81,48	79,94	80,46	79,50	79,16	81,85	79,28	82,05	81,25	80,48	78,39	79,46	80,97	80,26
1950-1959	80,82	79,02	80,92	79,57	78,30	79,59	78,98	79,16	79,26	81,48	80,20	80,06	78,91	79,48	81,21	79,80
1960-1969	80,81	79,90	81,82	80,05	78,66	80,66	79,58	80,15	80,22	81,01	80,55	80,02	80,02	80,58	81,89	80,39
1970-1979	82,13	80,07	83,49	81,47	78,95	81,59	80,60	80,64	81,10	81,18	80,89	81,12	80,72	81,63	82,98	81,24
1980-1989	83,37	81,15	84,33	82,77	80,21	82,49	81,49	81,67	82,51	82,12	81,64	82,31	82,00	82,21	84,32	82,31
1990-1999	83,82	83,47	84,68	83,91	82,21	84,49	83,18	83,52	83,80	83,41	83,25	83,61	83,97	83,66	85,01	83,73
2000-2007	84,87	84,79	85,24	85,14	84,07	86,36	85,15	85,78	84,46	84,39	84,45	84,89	85,36	84,99	85,53	85,03

The next Figure 7 illustrates the graphs from the DTR application to Sweden (females) for the period 1751-1759 (red line) and 1950-1959 (brown line). The bleu line expresses the life expectancy estimates for 2000-2008 downloaded from the human mortality database. Our application includes the periods 1800-1809, 1850-1859 and 2000-2008. The results of the DTR method refer to the future limits of the life expectancy in connection to the limits of the human organism and the developments of science. Our results for 2000-2008 suggest a future level for the life expectancy at birth in Sweden at 84,89 years, 2,34 years higher than the period 2000-2008 (see Table V).

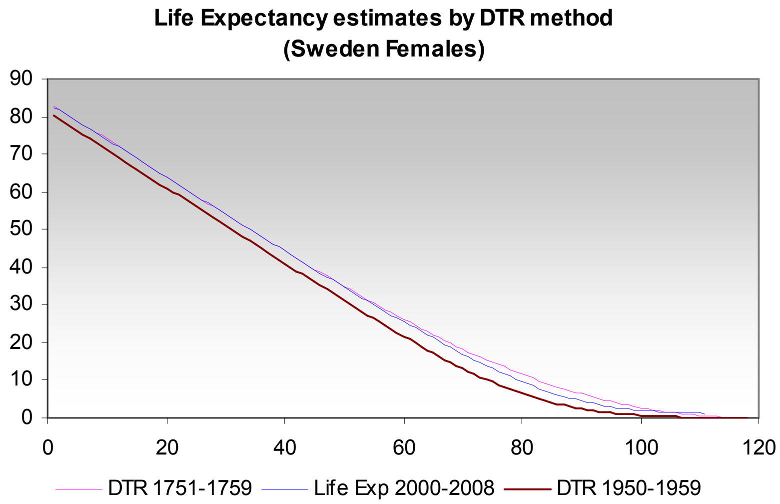

Fig. 7. DTR application to Sweden (females) for the period 1751-1759 (red line) and 1950-1959 (brown line). Life expectancy estimates for 2000-2008 are provided from the human mortality database (bleu line).

Estimation of the life expectancy by the DTR system

The estimation of the life expectancy by using the DTR system is presented in Table IV. In the first column the data for the deaths per age are presented. In our example the data for the period 1751-1759 for Sweden (females) are included. In the second column the data are normalized divided by their sum. This is important in order to compare data sets from various periods and from several countries. The next step is to apply the first exit time model including infant mortality (IM-model) to the normalized data. The fitting estimates appear in the third column. The fourth column is the age in years. The next step is to estimate the Deterioration Function K(x). We need the values for the parameter *b* and the exponent *c* already estimated in the previous step when applying the IM-model. The deterioration function values are normalized and included in the sixth column. Then the DTR values are estimated in the seventh column. These values are computed as a multiplication: $xK(x)$ and then are normalized and stored in the eight column. Then the DTR survival curve is estimated and included in column nine. We apply the following formula for the survival curve (SC): $SC_x = 1 - \sum_0^{x-1} xK(x)$.

The final step is the formulation of the tenth column including the DTR life expectancy list according to age from the survival curve. The life expectancy e_x at age x is calculated by: $e_x = \sum_0^n SC_x - \sum_0^x SC_x$, where n was selected the age 117 as a level according to our experience. However, the selection of a different age level will change the estimates for the life expectancy. The best approach will be the selection of a level age which is in accordance to reality.

The results of the application of the DTR method for various time periods in Sweden for females are summarized in Table V. The estimated life expectancy values are very close to that achieved in recent years as is also illustrated in Figure 8 thus supporting our argument for using the DTR method to forecasts the future trends of life expectancy. Even from the period 1751-1759 we can estimate a life expectancy at birth at the level of 82.92 years for Sweden, females. This value is very close to that of the period 2000-2008 (82.55 years).

TABLE IV

DTR System (Sweden, Females)

1751-1759 Data	Data Normalised	IM Model Estimates	Year x	K(x)	K(x) Normalised	xK(x)	xK(x) Normalised	Survival curve	Life Expectancy
57425	0,2473	0,2317	0	4,56E-05	6,004E-05	0	0	1	82,92
14438	0,0622	0,0873	1	0,000112	0,0001478	0,000148	2,044E-06	1	81,92
9467	0,0408	0,0486	2	0,00019	0,0002505	0,000501	6,926E-06	1	80,92
6739	0,0290	0,0319	3	0,000276	0,000364	0,001092	1,51E-05	0,99999	79,92
4918	0,0212	0,0231	4	0,000369	0,0004866	0,001946	2,691E-05	0,99998	78,92
3619	0,0156	0,0177	5	0,000468	0,0006167	0,003083	4,264E-05	0,99995	77,92
2623	0,0113	0,0142	6	0,000572	0,0007535	0,004521	6,252E-05	0,99991	76,92
1875	0,0081	0,0117	7	0,00068	0,0008964	0,006275	8,676E-05	0,99984	75,92
1375	0,0059	0,0100	8	0,000793	0,0010447	0,008357	0,0001156	0,99976	74,92
1124	0,0048	0,0087	9	0,000909	0,001198	0,010782	0,0001491	0,99964	73,92
1078	0,0046	0,0077	10	0,001029	0,001356	0,01356	0,0001875	0,99949	72,92
1065	0,0046	0,0069	11	0,001152	0,0015184	0,016702	0,000231	0,9993	71,92
1044	0,0045	0,0063	12	0,001279	0,0016849	0,020218	0,0002796	0,99907	70,92
1013	0,0044	0,0058	13	0,001408	0,0018552	0,024117	0,0003335	0,99879	69,92
973	0,0042	0,0054	14	0,00154	0,0020292	0,028409	0,0003928	0,99846	68,92
929	0,0040	0,0051	15	0,001675	0,0022067	0,0331	0,0004577	0,99807	67,93
899	0,0039	0,0049	16	0,001812	0,0023875	0,038199	0,0005282	0,99761	66,93
888	0,0038	0,0047	17	0,001952	0,0025714	0,043714	0,0006045	0,99708	65,93
897	0,0039	0,0046	18	0,002093	0,0027584	0,049651	0,0006865	0,99648	64,93
926	0,0040	0,0045	19	0,002237	0,0029482	0,056016	0,0007746	0,99579	63,94
969	0,0042	0,0044	20	0,002384	0,0031408	0,062817	0,0008686	0,99502	62,94
1011	0,0044	0,0044	21	0,002532	0,0033361	0,070058	0,0009687	0,99415	61,95
1046	0,0045	0,0044	22	0,002682	0,0035338	0,077744	0,001075	0,99318	60,95
1076	0,0046	0,0044	23	0,002834	0,003734	0,085882	0,0011875	0,9921	59,96
1100	0,0047	0,0045	24	0,002987	0,0039364	0,094474	0,0013063	0,99092	58,97
1122	0,0048	0,0045	25	0,003143	0,0041411	0,103526	0,0014315	0,98961	57,97
1158	0,0050	0,0046	26	0,0033	0,0043477	0,113041	0,0015631	0,98818	56,99
1214	0,0052	0,0047	27	0,003458	0,0045563	0,123021	0,0017011	0,98662	56,00
1288	0,0055	0,0048	28	0,003618	0,0047668	0,13347	0,0018456	0,98491	55,01
1382	0,0060	0,0049	29	0,003779	0,0049789	0,144389	0,0019965	0,98307	54,03
1482	0,0064	0,0050	30	0,003941	0,0051927	0,15578	0,002154	0,98107	53,04
1537	0,0066	0,0051	31	0,004104	0,0054078	0,167643	0,0023181	0,97892	52,06
1535	0,0066	0,0053	32	0,004268	0,0056243	0,179979	0,0024887	0,9766	51,08
1476	0,0064	0,0054	33	0,004434	0,005842	0,192787	0,0026658	0,97411	50,11
1359	0,0059	0,0056	34	0,0046	0,0060608	0,206066	0,0028494	0,97145	49,13
1207	0,0052	0,0058	35	0,004766	0,0062804	0,219813	0,0030395	0,9686	48,16
1103	0,0047	0,0059	36	0,004934	0,0065007	0,234027	0,003236	0,96556	47,19
1068	0,0046	0,0061	37	0,005101	0,0067217	0,248702	0,0034389	0,96232	46,23
1104	0,0048	0,0063	38	0,005269	0,006943	0,263834	0,0036482	0,95888	45,26
1209	0,0052	0,0065	39	0,005437	0,0071645	0,279417	0,0038636	0,95523	44,31
1362	0,0059	0,0067	40	0,005605	0,0073861	0,295444	0,0040853	0,95137	43,35
1469	0,0063	0,0069	41	0,005774	0,0076075	0,311908	0,0043129	0,94729	42,40
1509	0,0065	0,0071	42	0,005941	0,0078286	0,328799	0,0045465	0,94297	41,45
1481	0,0064	0,0073	43	0,006109	0,008049	0,346107	0,0047858	0,93843	40,51
1386	0,0060	0,0076	44	0,006275	0,0082686	0,36382	0,0050307	0,93364	39,57
1245	0,0054	0,0078	45	0,006441	0,0084872	0,381925	0,0052811	0,92861	38,64
1144	0,0049	0,0080	46	0,006606	0,0087045	0,400408	0,0055367	0,92333	37,71
1106	0,0048	0,0082	47	0,00677	0,0089203	0,419253	0,0057972	0,91779	36,78
1129	0,0049	0,0084	48	0,006932	0,0091342	0,438443	0,0060626	0,91199	35,87
1215	0,0052	0,0087	49	0,007093	0,0093461	0,457959	0,0063324	0,90593	34,95
1346	0,0058	0,0089	50	0,007252	0,0095556	0,47778	0,0066065	0,8996	34,05
1450	0,0062	0,0091	51	0,007409	0,0097625	0,497886	0,0068845	0,89299	33,15
1509	0,0065	0,0093	52	0,007564	0,0099664	0,518253	0,0071662	0,88611	32,26
1524	0,0066	0,0095	53	0,007716	0,0101671	0,538855	0,007451	0,87894	31,37
1494	0,0064	0,0097	54	0,007866	0,0103642	0,559667	0,0077388	0,87149	30,49
1438	0,0062	0,0099	55	0,008012	0,0105575	0,58066	0,0080291	0,86375	29,62
1431	0,0062	0,0101	56	0,008156	0,0107465	0,601805	0,0083215	0,85572	28,76
1491	0,0064	0,0102	57	0,008296	0,010931	0,62307	0,0086155	0,8474	27,90
1619	0,0070	0,0104	58	0,008432	0,0111107	0,644423	0,0089108	0,83879	27,05
1814	0,0078	0,0106	59	0,008565	0,0112852	0,66583	0,0092068	0,82988	26,21
2053	0,0088	0,0107	60	0,008693	0,0114543	0,687255	0,009503	0,82067	25,38

2246	0,0097	0,0108	61	0,008817	0,0116174	0,708662	0,0097991	0,81117	24,56
2371	0,0102	0,0109	62	0,008936	0,0117744	0,730013	0,0100943	0,80137	23,75
2428	0,0105	0,0110	63	0,00905	0,0119249	0,751269	0,0103882	0,79127	22,95
2416	0,0104	0,0111	64	0,009159	0,0120686	0,77239	0,0106803	0,78088	22,16
2357	0,0102	0,0111	65	0,009263	0,0122052	0,793336	0,0109699	0,7702	21,38
2343	0,0101	0,0112	66	0,009361	0,0123343	0,814064	0,0112565	0,75923	20,61
2395	0,0103	0,0112	67	0,009453	0,0124557	0,834534	0,0115395	0,74798	19,85
2512	0,0108	0,0111	68	0,009539	0,0125692	0,854702	0,0118184	0,73644	19,10
2696	0,0116	0,0111	69	0,009619	0,0126743	0,874527	0,0120926	0,72462	18,36
2918	0,0126	0,0110	70	0,009692	0,0127709	0,893965	0,0123613	0,71253	17,64
3060	0,0132	0,0109	71	0,009759	0,0128588	0,912975	0,0126242	0,70017	16,93
3096	0,0133	0,0108	72	0,009819	0,0129377	0,931515	0,0128806	0,68754	16,23
3025	0,0130	0,0107	73	0,009872	0,0130074	0,949543	0,0131298	0,67466	15,54
2846	0,0123	0,0105	74	0,009917	0,0130678	0,967019	0,0133715	0,66153	14,87
2588	0,0111	0,0103	75	0,009956	0,0131187	0,983905	0,013605	0,64816	14,20
2358	0,0102	0,0101	76	0,009987	0,01316	1,000161	0,0138298	0,63455	13,56
2184	0,0094	0,0098	77	0,010011	0,0131916	1,015751	0,0140453	0,62072	12,92
2065	0,0089	0,0096	78	0,010028	0,0132133	1,030641	0,0142512	0,60668	12,30
2003	0,0086	0,0093	79	0,010037	0,0132253	1,044797	0,014447	0,59243	11,69
1979	0,0085	0,0089	80	0,010039	0,0132273	1,058188	0,0146321	0,57798	11,10
1927	0,0083	0,0086	81	0,010033	0,0132196	1,070785	0,0148063	0,56335	10,52
1828	0,0079	0,0082	82	0,010019	0,013202	1,082563	0,0149692	0,54854	9,96
1683	0,0072	0,0079	83	0,009999	0,0131747	1,093498	0,0151204	0,53357	9,41
1492	0,0064	0,0075	84	0,009971	0,0131377	1,103567	0,0152596	0,51845	8,88
1267	0,0055	0,0071	85	0,009935	0,0130912	1,112754	0,0153866	0,50319	8,36
1061	0,0046	0,0067	86	0,009893	0,0130354	1,121042	0,0155013	0,48781	7,86
887	0,0038	0,0062	87	0,009844	0,0129703	1,12842	0,0156033	0,47231	7,37
743	0,0032	0,0058	88	0,009787	0,0128963	1,134876	0,0156925	0,4567	6,90
631	0,0027	0,0054	89	0,009725	0,0128136	1,140407	0,015769	0,44101	6,44
586	0,0025	0,0050	90	0,009655	0,0127223	1,145007	0,0158326	0,42524	6,00
484	0,0021	0,0046	91	0,00958	0,0126228	1,148676	0,0158834	0,40941	5,57
395	0,0017	0,0042	92	0,009498	0,0125154	1,151418	0,0159213	0,39352	5,16
318	0,0014	0,0038	93	0,009411	0,0124004	1,153238	0,0159464	0,3776	4,77
252	0,0011	0,0034	94	0,009318	0,0122781	1,154144	0,015959	0,36166	4,39
197	0,0008	0,0031	95	0,00922	0,0121489	1,154147	0,015959	0,3457	4,03
151	0,0007	0,0028	96	0,009117	0,0120131	1,153262	0,0159468	0,32974	3,68
115	0,0005	0,0024	97	0,009009	0,0118712	1,151505	0,0159225	0,31379	3,36
86	0,0004	0,0021	98	0,008897	0,0117234	1,148894	0,0158864	0,29787	3,04
63	0,0003	0,0019	99	0,008781	0,0115702	1,145451	0,0158388	0,28198	2,74
45	0,0002	0,0016	100	0,008661	0,011412	1,141199	0,01578	0,26614	2,46
32	0,0001	0,0014	101	0,008537	0,0112491	1,136163	0,0157103	0,25036	2,20
23	0,0001	0,0012	102	0,00841	0,0110821	1,13037	0,0156302	0,23465	1,95
15	0,0001	0,0010	103	0,008281	0,0109111	1,123847	0,01554	0,21902	1,71
10	0,0000	0,0008	104	0,008148	0,0107368	1,116625	0,0154402	0,20348	1,49
6	0,0000	0,0007	105	0,008014	0,0105594	1,108734	0,0153311	0,18804	1,29
3	0,0000	0,0006	106	0,007877	0,0103793	1,100205	0,0152131	0,17271	1,10
1	0,0000	0,0005	107	0,007739	0,0101969	1,091072	0,0150868	0,1575	0,93
0	0,0000	0,0004	108	0,007599	0,0100127	1,081367	0,0149526	0,14241	0,77
0	0,0000	0,0003	109	0,007458	0,0098268	1,071124	0,014811	0,12746	0,63
0	0,0000	0,0002	110	0,007316	0,0096398	1,060376	0,0146624	0,11265	0,50
	0,0000	0,0002	111	0,007173	0,0094519	1,049157	0,0145073	0,09799	0,39
	0,0000	0,0001	112	0,00703	0,0092634	1,037502	0,0143461	0,08348	0,29
	0,0000	0,0001	113	0,006887	0,0090747	1,025442	0,0141793	0,06913	0,21
	0,0000	0,0001	114	0,006744	0,0088861	1,013012	0,0140075	0,05495	0,14
	0,0000	0,0001	115	0,006601	0,0086978	1,000245	0,0138309	0,04095	0,08
	0,0000	0,0000	116	0,006459	0,0085101	0,987171	0,0136501	0,02712	0,04
	0,0000	0,0000	117	0,006317	0,0083233	0,973824	0,0134656	0,01347	0,01

TABLE V

Results from the DTR System for the Future Life Expectancy in Sweden (Females)

Year x	1751-1759	1800-1809	1850-1859	1900-1909	1950-1959	2000-2008	Life Exp 2000-2008
0	82,92	81,21	80,76	80,42	80,06	84,89	82,55
1	81,92	80,21	79,76	79,42	79,06	83,89	81,77
2	80,92	79,21	78,76	78,42	78,06	82,89	80,79
3	79,92	78,21	77,76	77,42	77,06	81,89	79,8
4	78,92	77,21	76,76	76,42	76,06	80,89	78,81
5	77,92	76,21	75,76	75,42	75,06	79,89	77,82
6	76,92	75,21	74,76	74,42	74,06	78,89	76,83
7	75,92	74,21	73,76	73,42	73,06	77,89	75,84
8	74,92	73,21	72,76	72,42	72,06	76,89	74,84
9	73,92	72,21	71,76	71,42	71,06	75,89	73,85
10	72,92	71,21	70,76	70,42	70,06	74,89	72,85
11	71,92	70,21	69,76	69,42	69,06	73,89	71,86
12	70,92	69,21	68,76	68,42	68,06	72,89	70,86
13	69,92	68,21	67,76	67,42	67,06	71,89	69,87
14	68,92	67,22	66,76	66,42	66,06	70,89	68,88
15	67,93	66,22	65,76	65,42	65,06	69,89	67,88
16	66,93	65,22	64,76	64,42	64,06	68,89	66,9
17	65,93	64,22	63,77	63,42	63,06	67,89	65,91
18	64,93	63,22	62,77	62,42	62,06	66,89	64,92
19	63,94	62,22	61,77	61,42	61,06	65,89	63,94
20	62,94	61,23	60,77	60,42	60,06	64,89	62,96
21	61,95	60,23	59,77	59,42	59,06	63,89	61,97
22	60,95	59,24	58,78	58,42	58,07	62,89	60,99
23	59,96	58,24	57,78	57,42	57,07	61,89	60
24	58,97	57,25	56,79	56,42	56,07	60,89	59,02
25	57,97	56,25	55,79	55,42	55,07	59,89	58,03
26	56,99	55,26	54,80	54,42	54,07	58,89	57,05
27	56,00	54,27	53,81	53,43	53,07	57,89	56,06
28	55,01	53,28	52,82	52,43	52,07	56,89	55,08
29	54,03	52,30	51,83	51,43	51,07	55,89	54,09
30	53,04	51,31	50,84	50,43	50,07	54,89	53,11
31	52,06	50,33	49,86	49,43	49,07	53,89	52,12
32	51,08	49,35	48,87	48,44	48,07	52,89	51,14
33	50,11	48,37	47,89	47,44	47,07	51,89	50,16
34	49,13	47,39	46,91	46,44	46,07	50,89	49,17
35	48,16	46,41	45,93	45,45	45,08	49,89	48,19
36	47,19	45,44	44,96	44,46	44,08	48,89	47,21
37	46,23	44,47	43,98	43,46	43,08	47,89	46,24
38	45,26	43,51	43,02	42,47	42,09	46,89	45,26
39	44,31	42,55	42,05	41,48	41,09	45,89	44,29
40	43,35	41,59	41,09	40,49	40,10	44,89	43,31
41	42,40	40,63	40,13	39,51	39,11	43,89	42,34
42	41,45	39,68	39,17	38,52	38,11	42,89	41,37
43	40,51	38,74	38,22	37,54	37,12	41,89	40,41
44	39,57	37,79	37,28	36,56	36,13	40,89	39,45
45	38,64	36,86	36,34	35,58	35,15	39,89	38,49
46	37,71	35,93	35,40	34,60	34,16	38,89	37,53
47	36,78	35,00	34,47	33,63	33,18	37,90	36,58
48	35,87	34,08	33,55	32,66	32,20	36,90	35,64
49	34,95	33,17	32,63	31,70	31,22	35,90	34,7
50	34,05	32,26	31,72	30,74	30,25	34,91	33,76
51	33,15	31,36	30,81	29,78	29,28	33,91	32,83
52	32,26	30,47	29,91	28,83	28,31	32,92	31,91
53	31,37	29,58	29,02	27,89	27,35	31,92	30,98
54	30,49	28,71	28,14	26,95	26,39	30,93	30,07
55	29,62	27,84	27,27	26,02	25,44	29,94	29,15
56	28,76	26,98	26,40	25,10	24,50	28,95	28,24
57	27,90	26,12	25,55	24,18	23,56	27,97	27,35
58	27,05	25,28	24,70	23,27	22,63	26,98	26,46
59	26,21	24,45	23,86	22,37	21,71	26,00	25,57
60	25,38	23,62	23,04	21,48	20,79	25,02	24,7

61	24,56	22,81	22,22	20,61	19,89	24,05	23,82
62	23,75	22,01	21,42	19,74	19,00	23,07	22,96
63	22,95	21,22	20,63	18,88	18,11	22,11	22,1
64	22,16	20,44	19,85	18,04	17,25	21,15	21,24
65	21,38	19,67	19,08	17,21	16,39	20,19	20,4
66	20,61	18,91	18,32	16,40	15,55	19,24	19,57
67	19,85	18,17	17,58	15,60	14,73	18,30	18,74
68	19,10	17,44	16,85	14,82	13,92	17,37	17,92
69	18,36	16,72	16,13	14,06	13,13	16,45	17,11
70	17,64	16,02	15,43	13,31	12,36	15,54	16,32
71	16,93	15,33	14,74	12,58	11,61	14,65	15,53
72	16,23	14,65	14,07	11,87	10,88	13,76	14,76
73	15,54	13,99	13,41	11,18	10,18	12,89	14
74	14,87	13,34	12,77	10,52	9,50	12,04	13,24
75	14,20	12,70	12,15	9,87	8,84	11,21	12,51
76	13,56	12,09	11,53	9,25	8,21	10,40	11,78
77	12,92	11,48	10,94	8,65	7,61	9,62	11,08
78	12,30	10,89	10,36	8,07	7,03	8,86	10,4
79	11,69	10,32	9,80	7,52	6,48	8,12	9,73
80	11,10	9,77	9,25	6,99	5,96	7,42	9,1
81	10,52	9,22	8,72	6,48	5,47	6,75	8,48
82	9,96	8,70	8,21	6,00	5,00	6,11	7,89
83	9,41	8,19	7,72	5,54	4,56	5,50	7,33
84	8,88	7,70	7,24	5,11	4,15	4,94	6,79
85	8,36	7,22	6,78	4,70	3,77	4,41	6,28
86	7,86	6,76	6,33	4,31	3,42	3,92	5,8
87	7,37	6,32	5,90	3,94	3,09	3,47	5,36
88	6,90	5,89	5,49	3,60	2,78	3,05	4,94
89	6,44	5,48	5,10	3,28	2,50	2,67	4,55
90	6,00	5,08	4,72	2,98	2,24	2,33	4,2
91	5,57	4,70	4,36	2,70	2,00	2,03	3,87
92	5,16	4,34	4,01	2,43	1,78	1,76	3,58
93	4,77	3,99	3,68	2,19	1,59	1,51	3,3
94	4,39	3,66	3,37	1,97	1,40	1,30	3,04
95	4,03	3,35	3,07	1,76	1,24	1,11	2,81
96	3,68	3,05	2,79	1,57	1,09	0,95	2,6
97	3,36	2,76	2,53	1,39	0,96	0,81	2,41
98	3,04	2,49	2,28	1,23	0,84	0,68	2,25
99	2,74	2,24	2,04	1,08	0,73	0,57	2,09
100	2,46	2,00	1,82	0,95	0,63	0,48	1,96
101	2,20	1,78	1,61	0,82	0,54	0,40	1,84
102	1,95	1,57	1,42	0,71	0,46	0,33	1,74
103	1,71	1,37	1,24	0,61	0,39	0,28	1,64
104	1,49	1,19	1,07	0,52	0,33	0,23	1,56
105	1,29	1,03	0,92	0,44	0,28	0,18	1,49
106	1,10	0,87	0,78	0,37	0,23	0,15	1,43
107	0,93	0,73	0,66	0,30	0,19	0,12	1,37
108	0,77	0,61	0,54	0,24	0,15	0,09	1,33
109	0,63	0,49	0,44	0,19	0,12	0,07	1,29
110	0,50	0,39	0,35	0,15	0,09	0,05	1,27
111	0,39	0,30	0,27	0,11	0,07	0,04	
112	0,29	0,22	0,20	0,08	0,05	0,03	
113	0,21	0,16	0,14	0,06	0,03	0,02	
114	0,14	0,10	0,09	0,04	0,02	0,01	
115	0,08	0,06	0,05	0,02	0,01	0,01	
116	0,04	0,03	0,03	0,01	0,01	0,00	
117	0,01	0,01	0,01	0,00	0,00	0,00	

Another indicator is the estimated maximum deterioration age (Max Det) which is stable and independent of the age level selected. The maximum deterioration age during recent years tends to coincide with the life expectancy at birth estimated with the classical techniques of constructing life tables. Both the DTR and the Max Det are quite good measures of the life expectancy now and in the future as is presented in Figure 8 where the mean values for the 15 countries studied are given.

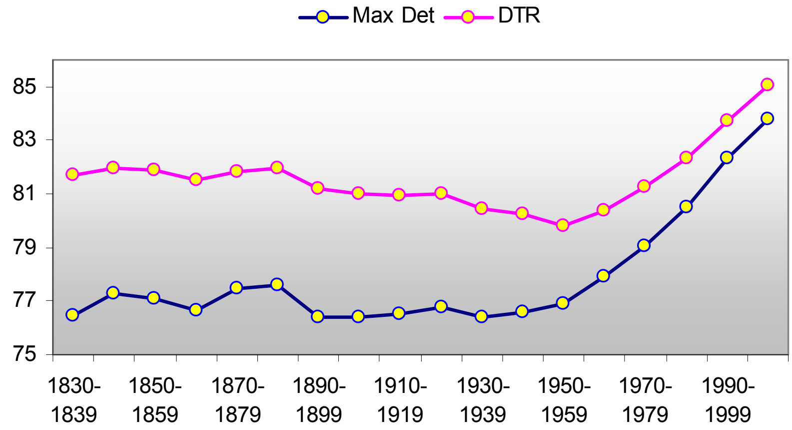

Fig. 8. Maximum deterioration age and DTR for 15 Countries

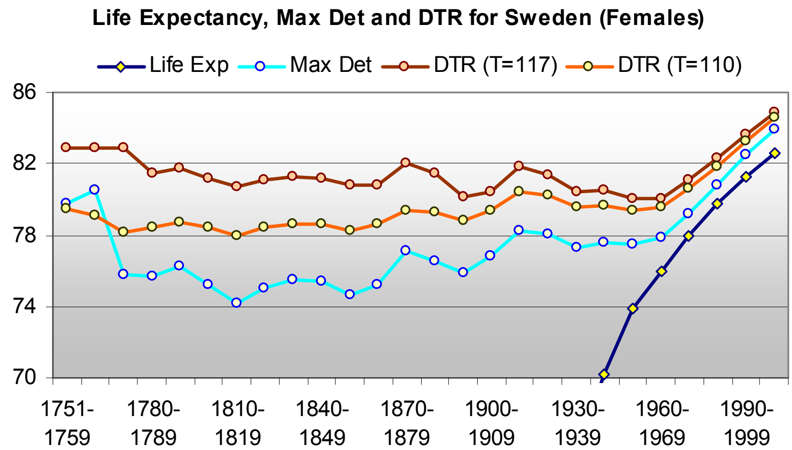

Figure 9. Life expectancy, Maximum deterioration age and DTR for Sweden, females

The influence of the life level T for the estimation of life expectancy by the DTR system in Sweden (females) is illustrated in Figure 9. Two scenarios are selected for the estimation of the future life expectancy at birth. In the first a T=117 year level is accepted (dark brawn line) and in the second T=110. As it was expected the higher level for T suggests a higher level for the life expectancy at birth via the DTR method. However, both scenarios tend to coincide in recent years something that it is quite useful in estimating the future trends for the life expectancy development (see Table VI). That it is important with the DTR system is that we can construct life tables for future dates thus doing forecasts. Instead with the Max Det we can have only an estimate for the future levels of life expectancy at birth but not for the life expectancy in other ages. The standard life expectancy at birth is presented (blue line) and the Max Deterioration points are also presented (light bleu line).

TABLE VI

Year	Max Det	DTR
1830-1839	76,47	81,67
1840-1849	77,26	81,98
1850-1859	77,10	81,89
1860-1869	76,65	81,51
1870-1879	77,49	81,83
1880-1889	77,59	81,96
1890-1899	76,38	81,17
1900-1909	76,39	81,03
1910-1919	76,52	80,93
1920-1929	76,76	80,98
1930-1939	76,36	80,42
1940-1949	76,57	80,26
1950-1959	76,87	79,80
1960-1969	77,90	80,39
1970-1979	79,02	81,24
1980-1989	80,50	82,31
1990-1999	82,32	83,73
2000-2007	83,81	85,03

The Halley Life Table

Edmund Halley published his famous paper in 1693. It was a pioneering study indicating of how a scientist of a high calibre could cope to a precisely selected data sets. Halley realized that to construct a life table from only mortality data was fusible only on the basis of a stationary population (Keyfitz and Caswell, 1977) by means of a population where births and deaths are almost equal and the incoming and outgoing people are limited. This was the case of the Breslau city in Silesia (now Wroclaw). The birth and death data sent to Halley gave him the opportunity to construct a life table and present his results in the paper on "An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslau; with an Attempt to ascertain the Price of Annuities upon Lives". The same time period was also proposed a method for handling life tables by Graunt (1662). For more information on the history and the development of actuarial science see the related history by Haberman and Sibbett (1995).

The purpose of this chapter is first to use the Halley's life table data in order to construct a mortality curve by applying a stochastic model resulting from the first exit time theory. After applying the model and constructing the mortality curve we find the deterioration function for the specific population of Breslau at the years studied by Halley and thus making possible to find the maximum deterioration age and constructing a graph for the "vitality" of the population, a term proposed by Halley and used many years later by Strehler and Mildvan (1960) who also suggest the term "vitality" of a person in a stochastic modeling of the human life.

In Table VII the first two columns include the Breslau life table data from the Halley paper whereas in the third column we have constructed the deaths per age as the difference between two consecutive rows of the second column. The data from the third column are inserted into our Excel program of the fist exit time distribution function and the results are presented in Figure 10.

TABLE VII

Age	Persons (Total)	Persons	Age	Persons (Total)	Persons
1	1000	145	51	335	11
2	855	57	52	324	11
3	798	38	53	313	11
4	760	28	54	302	10
5	732	22	55	292	10
6	710	18	56	282	10
7	692	12	57	272	10
8	680	10	58	262	10
9	670	9	59	252	10
10	661	8	60	242	10
11	653	7	61	232	10
12	646	6	62	222	10
13	640	6	63	212	10
14	634	6	64	202	10
15	628	6	65	192	10
16	622	6	66	182	10
17	616	6	67	172	10
18	610	6	68	162	10
19	604	6	69	152	10
20	598	6	70	142	11
21	592	6	71	131	11
22	586	7	72	120	11
23	579	6	73	109	11
24	573	6	74	98	10
25	567	7	75	88	10
26	560	7	76	78	10
27	553	7	77	68	10
28	546	7	78	58	9
29	539	8	79	49	8
30	531	8	80	41	7
31	523	8	81	34	6
32	515	8	82	28	5
33	507	8	83	23	3
34	499	9	84	20	
35	490	9	85		
36	481	9	86		
37	472	9	87		
38	463	9	88		
39	454	9	89		
40	445	9	90		
41	436	9	91		
42	427	10	92		
43	417	10	93		
44	407	10	94		
45	397	10	95		
46	387	10	96		
47	377	10	97		
48	367	10	98		
49	357	11	99		
50	346	11	100		

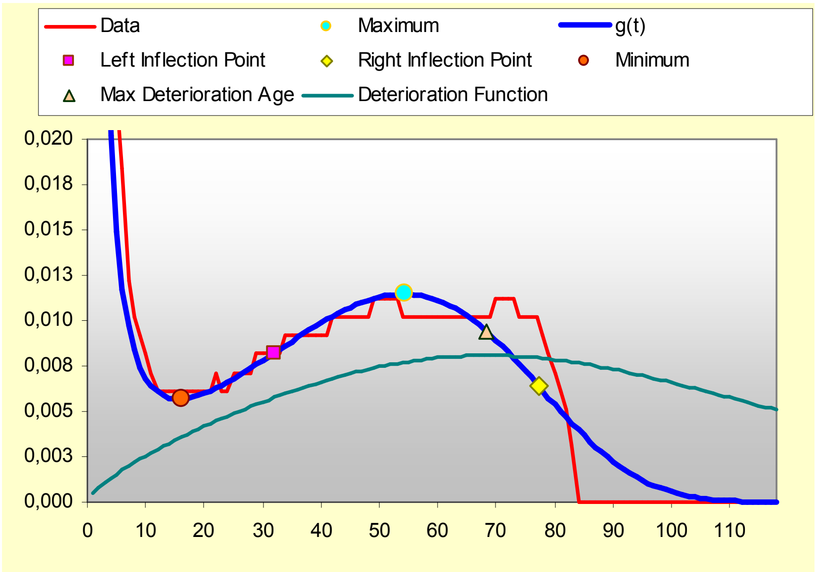

Fig. 10. Fit curve, data plot and deterioration curve for Halley data

The estimated best fit is presented with a blue line. The parameter estimates and the values for the characteristic points are given in the next Table VIII:

TABLE VIII

Characteristic Points of Graph	Year	g(t)	g'(t)	Parameter
Maximum	53,3	0,011457	0	c = 2,72
Left Inflection Point	30,9	0,008182	0,0001081	b = 0,03425
Right Inflection Point	76,2	0,006412	-0,000470	l = 0,25907
Minimum	14,6	0,005679	0	k = 0,61215
Maximum Deterioration Age	67,4			

From the previous Figure and Table the estimated maximum death rate is at the age of 53,3 years, the right inflection point is at 76,2 years, the left inflection point is at 30,9 years and the minimum at 14,6 years. A very important characteristic of the health state of the population is given by estimating the age where the maximum deterioration takes place. This is estimated at the age of 67,39 years and it is the maximum of the deterioration function presented by a green curve in the graph.

TABLE IX

DTR System (Halley, Breslaw Data)

1687-1691 Data	Data Normalised	IM Model Estimates	Year x	K(x)	K(x) Normalised	xK(x)	xK(x) Normalised	Survival curve	Life Expectancy
145	0,1480	0,1535	0	0,0004834	0,00067177	0	0	1	79,07
57	0,0582	0,0554	1	0,0007963	0,00110653	0,0007963	1,705E-05	1	78,07
38	0,0388	0,0306	2	0,0010662	0,00148166	0,0021325	4,566E-05	0,99998295	77,07
28	0,0286	0,0202	3	0,0013116	0,00182264	0,0039348	8,4251E-05	0,99993729	76,07
22	0,0224	0,0149	4	0,0015402	0,00214027	0,0061607	0,00013191	0,99985304	75,07
18	0,0184	0,0117	5	0,0017562	0,00244044	0,0087809	0,00018802	0,99972113	74,07
12	0,0122	0,0097	6	0,0019623	0,0027268	0,0117735	0,00025209	0,99953311	73,07
10	0,0102	0,0084	7	0,0021602	0,00300181	0,0151211	0,00032377	0,99928102	72,07
9	0,0092	0,0074	8	0,0023512	0,00326723	0,0188093	0,00040274	0,99895725	71,07
8	0,0082	0,0068	9	0,0025362	0,00352437	0,0228259	0,00048874	0,99855451	70,07
7	0,0071	0,0064	10	0,002716	0,00377424	0,0271602	0,00058155	0,99806577	69,07
6	0,0061	0,0061	11	0,0028912	0,00401762	0,0318027	0,00068095	0,99748422	68,08
6	0,0061	0,0059	12	0,0030621	0,00425514	0,0367449	0,00078677	0,99680327	67,08
6	0,0061	0,0057	13	0,0032291	0,0044873	0,0419789	0,00089884	0,99601649	66,08
6	0,0061	0,0057	14	0,0033927	0,00471452	0,0474973	0,001017	0,99511765	65,09
6	0,0061	0,0057	15	0,0035529	0,00493715	0,0532931	0,0011411	0,99410065	64,09
6	0,0061	0,0057	16	0,00371	0,00515547	0,0593596	0,00127099	0,99295955	63,10
6	0,0061	0,0058	17	0,0038641	0,0053697	0,0656904	0,00140655	0,99168856	62,10
6	0,0061	0,0059	18	0,0040155	0,00558005	0,0722792	0,00154763	0,99028201	61,11
6	0,0061	0,0060	19	0,0041642	0,00578667	0,0791198	0,00169409	0,98873439	60,12
6	0,0061	0,0061	20	0,0043103	0,0059897	0,086206	0,00184582	0,98704029	59,13
7	0,0071	0,0063	21	0,0044539	0,00618923	0,0935316	0,00200268	0,98519447	58,15
6	0,0061	0,0064	22	0,004595	0,00638535	0,1010905	0,00216452	0,98319179	57,16
6	0,0061	0,0066	23	0,0047337	0,00657812	0,1088761	0,00233123	0,98102727	56,18
7	0,0071	0,0068	24	0,0048701	0,0067676	0,1168823	0,00250266	0,97869604	55,20
7	0,0071	0,0070	25	0,0050041	0,00695381	0,1251023	0,00267866	0,97619338	54,22
7	0,0071	0,0072	26	0,0051357	0,00713676	0,1335295	0,0028591	0,97351472	53,24
7	0,0071	0,0074	27	0,0052651	0,00731647	0,1421569	0,00304383	0,97065562	52,27
8	0,0082	0,0076	28	0,0053921	0,00749292	0,1509774	0,00323269	0,96761179	51,30
8	0,0082	0,0078	29	0,0055167	0,00766611	0,1599837	0,00342553	0,96437909	50,33
8	0,0082	0,0080	30	0,0056389	0,007836	0,1691681	0,00362219	0,96095356	49,37
8	0,0082	0,0082	31	0,0057588	0,00800257	0,1785229	0,00382249	0,95733137	48,40
8	0,0082	0,0084	32	0,0058762	0,00816577	0,1880398	0,00402626	0,95350888	47,45
9	0,0092	0,0086	33	0,0059912	0,00832555	0,1977106	0,00423333	0,94948262	46,49
9	0,0092	0,0088	34	0,0061037	0,00848187	0,2075264	0,00444351	0,94524929	45,54
9	0,0092	0,0091	35	0,0062137	0,00863466	0,2174784	0,0046566	0,94080578	44,60
9	0,0092	0,0093	36	0,006321	0,00878386	0,2275573	0,0048724	0,93614918	43,66
9	0,0092	0,0095	37	0,0064258	0,00892941	0,2377536	0,00509072	0,93127678	42,72
9	0,0092	0,0097	38	0,0065278	0,00907122	0,2480573	0,00531134	0,92618605	41,79
9	0,0092	0,0099	39	0,0066271	0,00920923	0,2584584	0,00553405	0,92087471	40,86
9	0,0092	0,0101	40	0,0067237	0,00934336	0,2689465	0,00575862	0,91534066	39,94
10	0,0102	0,0102	41	0,0068173	0,00947353	0,2795107	0,00598482	0,90958204	39,03
10	0,0102	0,0104	42	0,0069081	0,00959966	0,2901403	0,00621242	0,90359722	38,12
10	0,0102	0,0106	43	0,0069959	0,00972167	0,3008238	0,00644117	0,89738481	37,22
10	0,0102	0,0107	44	0,0070807	0,00983948	0,3115499	0,00667083	0,89094364	36,32
10	0,0102	0,0109	45	0,0071624	0,009953	0,3223068	0,00690116	0,88427281	35,43
10	0,0102	0,0110	46	0,0072409	0,01006216	0,3330826	0,00713189	0,87737165	34,54
10	0,0102	0,0111	47	0,0073163	0,01016688	0,3438651	0,00736276	0,87023976	33,67
11	0,0112	0,0112	48	0,0073884	0,01026706	0,3546421	0,00759351	0,862877	32,79
11	0,0112	0,0113	49	0,0074572	0,01036265	0,3654011	0,00782388	0,85528349	31,93
11	0,0112	0,0114	50	0,0075226	0,01045357	0,3761295	0,0080536	0,84745961	31,08

11	0,0112	0,0114	51	0,0075846	0,01053974	0,3868146	0,00828238	0,83940601	30,23
11	0,0112	0,0114	52	0,0076431	0,0106211	0,3974436	0,00850997	0,83112363	29,39
10	0,0102	0,0115	53	0,0076982	0,01069758	0,4080037	0,00873608	0,82261366	28,56
10	0,0102	0,0115	54	0,0077497	0,01076912	0,4184821	0,00896044	0,81387758	27,74
10	0,0102	0,0114	55	0,0077976	0,01083568	0,4288659	0,00918278	0,80491714	26,92
10	0,0102	0,0114	56	0,0078418	0,01089719	0,4391425	0,00940282	0,79573437	26,12
10	0,0102	0,0113	57	0,0078824	0,01095362	0,449299	0,00962028	0,78633155	25,32
10	0,0102	0,0112	58	0,0079194	0,01100493	0,459323	0,00983491	0,77671127	24,54
10	0,0102	0,0111	59	0,0079526	0,01105109	0,4692019	0,01004644	0,76687635	23,76
10	0,0102	0,0110	60	0,0079821	0,01109206	0,4789236	0,0102546	0,75682991	22,99
10	0,0102	0,0108	61	0,0080078	0,01112783	0,4884759	0,01045913	0,74657531	22,23
10	0,0102	0,0107	62	0,0080298	0,01115839	0,4978472	0,01065979	0,73611618	21,49
10	0,0102	0,0105	63	0,008048	0,01118373	0,5070259	0,01085632	0,7254564	20,75
10	0,0102	0,0103	64	0,0080625	0,01120386	0,5160009	0,01104849	0,71460008	20,03
10	0,0102	0,0100	65	0,0080733	0,01121878	0,5247614	0,01123607	0,70355158	19,31
10	0,0102	0,0098	66	0,0080803	0,01122851	0,5332968	0,01141883	0,69231552	18,61
10	0,0102	0,0095	67	0,0080835	0,01123308	0,5415973	0,01159655	0,68089669	17,92
10	0,0102	0,0092	68	0,0080831	0,01123251	0,5496531	0,01176904	0,66930014	17,24
11	0,0112	0,0089	69	0,0080791	0,01122686	0,5574552	0,0119361	0,6575311	16,57
11	0,0112	0,0086	70	0,0080714	0,01121615	0,5649951	0,01209754	0,645595	15,91
11	0,0112	0,0083	71	0,0080601	0,01120046	0,5722645	0,01225319	0,63349746	15,26
11	0,0112	0,0079	72	0,0080452	0,01117983	0,579256	0,01240289	0,62124426	14,63
10	0,0102	0,0076	73	0,0080269	0,01115435	0,5859626	0,01254649	0,60884137	14,01
10	0,0102	0,0072	74	0,0080051	0,01112408	0,5923778	0,01268385	0,59629488	13,40
10	0,0102	0,0069	75	0,0079799	0,01108912	0,5984959	0,01281485	0,58361102	12,80
10	0,0102	0,0065	76	0,0079515	0,01104955	0,6043116	0,01293938	0,57079617	12,22
9	0,0092	0,0061	77	0,0079197	0,01100547	0,6098205	0,01305733	0,55785679	11,65
8	0,0082	0,0057	78	0,0078849	0,01095698	0,6150184	0,01316863	0,54479946	11,09
7	0,0071	0,0054	79	0,0078469	0,01090418	0,619902	0,0132732	0,53163083	10,55
6	0,0061	0,0050	80	0,0078059	0,01084721	0,6244688	0,01337098	0,51835763	10,01
5	0,0051	0,0047	81	0,0077619	0,01078616	0,6287165	0,01346193	0,50498666	9,50
3	0,0031	0,0043	82	0,0077152	0,01072118	0,6326437	0,01354602	0,49152473	8,99
0	0,0000	0,0040	83	0,0076657	0,01065238	0,6362496	0,01362323	0,47797871	8,50
0	0,0000	0,0037	84	0,0076135	0,0105799	0,6395341	0,01369355	0,46435548	8,02
0	0,0000	0,0033	85	0,0075588	0,01050388	0,6424973	0,013757	0,45066193	7,56
0	0,0000	0,0030	86	0,0075016	0,01042445	0,6451404	0,01381359	0,43690493	7,11
0	0,0000	0,0028	87	0,0074421	0,01034175	0,6474649	0,01386337	0,42309133	6,67
0	0,0000	0,0025	88	0,0073804	0,01025594	0,6494729	0,01390636	0,40922797	6,25
0	0,0000	0,0022	89	0,0073165	0,01016716	0,651167	0,01394263	0,39532161	5,84
0	0,0000	0,0020	90	0,0072506	0,01007555	0,6525504	0,01397226	0,38137897	5,44
0	0,0000	0,0018	91	0,0071827	0,00998127	0,6536268	0,0139953	0,36740672	5,06
0	0,0000	0,0016	92	0,007113	0,00988446	0,6544002	0,01401186	0,35341142	4,69
0	0,0000	0,0014	93	0,0070417	0,00978527	0,6548754	0,01402204	0,33939955	4,34
0	0,0000	0,0012	94	0,0069687	0,00968386	0,6550573	0,01402593	0,32537751	4,00
0	0,0000	0,0010	95	0,0068942	0,00958038	0,6549513	0,01402366	0,31135158	3,68
0	0,0000	0,0009	96	0,0068184	0,00947496	0,6545632	0,01401535	0,29732792	3,36
0	0,0000	0,0008	97	0,0067412	0,00936777	0,6538991	0,01400113	0,28331257	3,07
0	0,0000	0,0007	98	0,0066629	0,00925894	0,6529656	0,01398114	0,26931143	2,78
0	0,0000	0,0006	99	0,0065835	0,00914863	0,6517693	0,01395553	0,25533029	2,51
0	0,0000	0,0005	100	0,0065032	0,00903696	0,6503172	0,01392444	0,24137476	2,26
0	0,0000	0,0004	101	0,0064219	0,00892409	0,6486166	0,01388802	0,22745032	2,02
0	0,0000	0,0003	102	0,0063399	0,00881014	0,6466749	0,01384645	0,2135623	1,79
0	0,0000	0,0003	103	0,0062573	0,00869526	0,6444997	0,01379988	0,19971585	1,58
0	0,0000	0,0002	104	0,006174	0,00857958	0,6420989	0,01374847	0,18591597	1,38
0	0,0000	0,0002	105	0,0060903	0,00846321	0,6394804	0,0136924	0,1721675	1,19
0	0,0000	0,0001	106	0,0060062	0,00834629	0,636652	0,01363184	0,1584751	1,02
0	0,0000	0,0001	107	0,0059217	0,00822894	0,6336221	0,01356697	0,14484325	0,86
0	0,0000	0,0001	108	0,005837	0,00811127	0,6303986	0,01349795	0,13127629	0,71
0	0,0000	0,0001	109	0,0057522	0,00799339	0,6269897	0,01342496	0,11777834	0,58
0	0,0000	0,0001	110	0,0056673	0,00787542	0,6234038	0,01334817	0,10435338	0,47
0	0,0000	0,0000	111	0,0055824	0,00775747	0,6196488	0,01326777	0,09100521	0,36
0	0,0000	0,0000	112	0,0054976	0,00763962	0,615733	0,01318393	0,07773744	0,27
0	0,0000	0,0000	113	0,005413	0,00752198	0,6116645	0,01309682	0,06455351	0,19
0	0,0000	0,0000	114	0,0053285	0,00740464	0,6074512	0,0130066	0,05145669	0,13
0	0,0000	0,0000	115	0,0052444	0,00728769	0,6031012	0,01291346	0,03845009	0,08
0	0,0000	0,0000	116	0,0051605	0,00717121	0,5986224	0,01281756	0,02553663	0,04
0	0,0000	0,0000	117	0,0050771	0,00705528	0,5940223	0,01271907	0,01271907	0,01
	1,0000			0,7196193	1	46,70	1	79,07	

The DTR system as presented earlier provides the last Life Table IX from which we can find a future life expectancy based on the deterioration function. The surprising result is that the estimated life expectancy is quite close to the values for Germany from 2000-2008 and higher than the related values for Poland (2005-2009) as provided by the Human Mortality Database. The estimates are presented in the next Figure 11 along with the estimates for the life expectancy estimated by the Breslau data (see the green line in the graph).

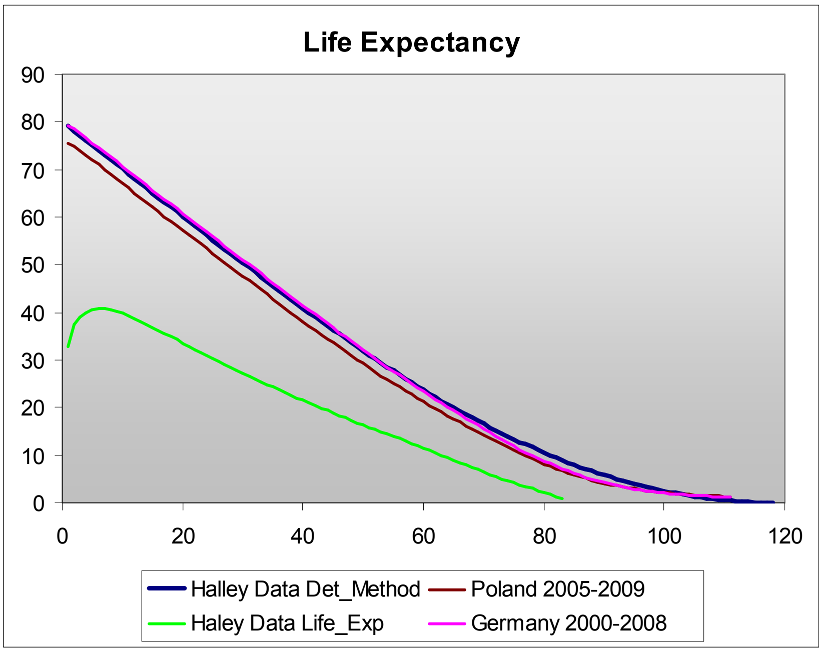

Fig. 11. Life expectancy curves for Breslau data

Illustration of the development of the maximum deterioration age from the Halley days until today is given in Figure 12. The maximum deterioration age for the Breslau data (1687-1691) was 67,39 years and continued to increase by 4,75 years per century.

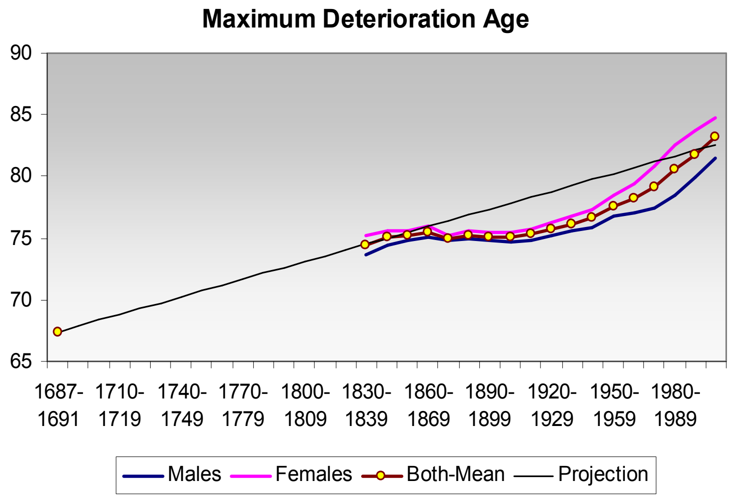

Fig. 12. Maximum Deterioration Age for various time periods

The Program

A computer program (IM-model-DTR-Life_Tables) was developed to be able to make the necessary computations related to this paper. Furthermore the program estimates the life expectancy tables by based on the mortality and population data. The life expectancy is also estimated by based on the fitting curve thus making more accurate the related estimations. The program and the related theory can be found in the website: http://www.cmsim.net. The program is developed in Excel 2003 and it is very easy to use without any special tool.

Conclusions

We have developed and applied a new theoretical framework for analyzing mortality data. The work starting several years ago was based on the stochastic theory and the derivation of a first exit time distribution function suitable for expressing the human mortality. Furthermore we have explored on how we could model the so-called "vitality" of a person or the opposite term the deterioration of an organism and to provide a function, the deterioration function, which could be useful for sociologists, police makers and the insurance people in making their estimates and plan the future.

Acknowledgments

The data used can be downloaded from the Human Mortality Database at: http://www.mortality.org or from the statistical year-books of the countries studied.

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